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QUANTITIES, UNITS AND SYMBOLS IN HEAT AND MASS TRANSFER

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Abstract—The International System of Units (SI) is not only based on the definitions of seven base units. but also on a set of relations between physical quantities. Principles of quantity calculus are discussed. It is shown that the relation between two arbitrary quantities X and Y must be of the form

$$Y = kX^c$$

with constant k and c. An exception must be made for dimensionless quantities. Examples are given. Now that the SI has been generally adopted, the time seems ripe for agreement on the use of quantity symbols in heat and mass transfer. Some thoughts on this subject are presented.

NOMENCLATURE

- a, parameter or constant [1];
- a, acceleration [$m \cdot s^{-2}$];
- A, surface area [m^2];
- A, parameter [$W \cdot m^{-2} \cdot K^{-1}$];
- b, parameter or constant [1];
- b, breadth [m];
- B, parameter [$W \cdot m^{-2} \cdot K^{-1}$];
- c, constant [1];
- d, diameter [m];
- E, kinetic energy [J];
- F, force [N];
- g, constant [1];
- k, constant;
- l, length [m];
- L, heat of vaporization [$J \cdot kg^{-1}$];
- m, mass [kg];
- m, exponent [1];
- M, molar mass [$kg \cdot mol^{-1}$];
- n, amount of substance [mol];
- Nu, Nusselt number [1];
- p, pressure [Pa];
- p, exponent [1];
- P, arbitrary quantity;
- Pr, Prandtl number [1];
- q, exponent [1];
- Q, arbitrary quantity;
- R, gas constant [$J \cdot mol^{-1} \cdot K^{-1}$];
- Re, Reynolds number [1];
- s, path length [m];
- S, sensitivity of hot wire [$J \cdot m^{-3} \cdot K^{-1}$];
- t, time [s];
- T, temperature [K];
- U, wind velocity [$m \cdot s^{-1}$];
- v, velocity [$m \cdot s^{-1}$];
- V, volume [m^3];
- W, work [J];
- X, arbitrary quantity;
- Y, arbitrary quantity.

Greek symbols

- α, heat-transfer coefficient [$W \cdot m^{-2} \cdot K^{-1}$];
- λ, multiplication factor [1];
- λ, thermal conductivity [$W \cdot m^{-1} \cdot K^{-1}$];
- ν, kinematic viscosity [$m^2 \cdot s^{-1}$].

1. INTRODUCTION

THOSE who have known Allan Ede will not be surprised to learn that he was a staunch promoter of unity in units and therefore of the application of the International System (SI). In a clear and concise Editorial Announcement, published in 1966 [1] he introduced the system to readers of this Journal. In his introduction he wrote that the SI "appears to have every prospect of being adopted throughout the world, and of eventually superseding all others, in every branch of science, engineering and commerce". The developments of the past ten years have fully confirmed the validity of his statement. He also showed foresight in predicting the introduction of the symbol K for the unit of temperature difference and of the pascal (symbol Pa) as the name for the SI-unit of pressure.

Because the SI has been adopted by all prominent international standardization organizations and SI-units have been or are about to be given legal force in many countries, I shall not dwell on the merits of the system. Allan Ede's introduction [1] can still be recommended to readers who are not or only superficially familiar with the SI. More comprehensive information is given by the International Organization for Standardization (ISO) in its publication 31 [2].

In this paper I shall concentrate on quantities and their symbols, rather than on units. It is probably not generally known that the SI rests not only on the definition of seven base units, but also on a comprehensive system of relations between quantities. In the field of electromagnetism, for instance, the SI is based on the so-called rationalized system of equations with four base quantities [2, Part V]. In Section 2 of this paper some principles of quantity calculus are dealt with and a number of examples, taken from or related to the field of heat and mass transfer, are discussed.

Now that agreement on units has been reached the time seems ripe for a drive towards a greater unity in the use of quantity symbols for heat and mass transfer. This is a matter of little scientific but of considerable practical interest. The lengthy nomenclature lists preceding many papers may serve to illustrate this point. International bodies in our field, in particular the International Centre for Heat and Mass Transfer and the Assembly for International Heat Transfer Conferences, but also the Editorial Board of this Journal can play an important role in promoting unity and consistency in the use of symbols for quantities. Some thoughts on the subject are presented in Section 3 of the present paper.

2. QUANTITY CALCULUS

2.1. *General principles*

A quantity is a physical concept that lends itself to measurement. This means that the ratio of the magnitudes of quantities of the same kind can be determined. Choosing a certain specimen as the unit the magnitudes of other quantities of the same kind can be expressed as a number, called the numerical value of the quantity. Thus one can write [2, Part 0] for an arbitrary quantity X:

$$X = \{X\} \cdot [X] \tag{2.1.1}$$

where $[X]$ denotes the unit and $\{X\}$ the numerical value. The quantity represents a real or imaginary physical entity, e.g. the length of a rectangular room. Therefore it does not depend on the choice of the unit. When a new unit is introduced that is a times the original one, where a is a pure number, the numerical value changes to $\{X\}/a$.

Two quantities of the same kind can in principle be added or subtracted. When they are expressed in the same unit one has:

$$X_3 = X_1 \pm X_2 \tag{2.1.2}$$

or

$$\{X_3\}[X] = (\{X_1\} \pm \{X_2\})[X]$$

and

$$\{X_3\} = \{X_1\} \pm \{X_2\}. \tag{2.1.3}$$

Equation (2.1.2) expresses a relation between quantities; such relations are called quantity equations. Equation (2.1.3) expresses a relation between numerical values and is called a numerical value equation.

Mathematical operations on numerical values present no specific problems. On the contrary it should be remembered that *a mathematical operation performed on quantities must always correspond with a physical operation*, be it sometimes an imaginary one. Two simple examples of addition operations are a series connection of electrical resistances and a parallel connection of electrical condensers. However, a simple physical addition operator cannot be constructed in all cases; the addition of two temperatures being a case in point.

Multiplication and division of quantities are also common operations. A simple example is the multiplication of the length, l, and the breadth, b, of a rectangle to find its surface area, A. Hence:

$$A = lb \tag{2.1.4}$$

or

$$\{A\}[A] = \{l\}[l]\{b\}[b] = \{l\}\{b\}[l]^2$$

from which one can deduce

$$\{A\} = \{l\}\{b\} \quad \text{and} \quad [A] = [l]^2. \tag{2.1.5}$$

The "physical" operation connected with $[l]\cdot[l]$ is the construction of a square with a side of unit length. A fairly complicated example of the definition of a division operation is the division of a quantity of heat, Q, by a temperature, T, which is based on the concept of a heat engine performing a Carnot cycle.

Relations between quantities of a different kind must, by their nature, have a simple form, as will be shown below [3]*.

Let an (empirical) relation exist between the numerical value of a quantity Y and that of a quantity X:

$$\{Y\} = f(\{X\}). \tag{2.1.6}$$

Then the ratio of two values of $\{Y\}$ must be independent of the choice of $[X]$. Changing the latter by a factor $1/\lambda$ one has:

$$\frac{\{Y\}_1}{\{Y\}_2} = \frac{f(\{X\}_1)}{f(\{X\}_2)} = \frac{f(\lambda\{X\}_1)}{f(\lambda\{X\}_2)} \tag{2.1.7}$$

which must hold for positive $\{Y\}$, $\{X\}$ and λ. Multiplication by $f(\lambda\{X\}_2)$ and differentiation (denoted by a prime) with respect to λ gives:

$$\frac{f(\{X\}_1)}{f(\{X\}_2)}\{X_2\}f'(\lambda\{X_2\}) = \{X\}_1 f'(\lambda\{X_1\}). \tag{2.1.8}$$

Rearranging and substitution of $\lambda = 1$ yields:

$$\frac{\{X\}_2 f'(\{X\}_2)}{f(\{X\}_2)} = \frac{\{X\}_1 f'(\{X\}_1)}{f(\{X\}_1)} = c \tag{2.1.9}$$

where c must be a constant, since $\{X\}_1$ and $\{X\}_2$ were chosen arbitrarily. Dropping the indices one finds after integration:

$$f(\{X\}) = k\{X\}^c \tag{2.1.10}$$

with a positive integration constant k. Defining

$$[Y] = [X]^c \tag{2.1.11}$$

one arrives at the quantity equation

$$Y = kX^c. \tag{2.1.12}$$

An important exception to this general form of a relation between two quantities must be made for the case that X is a dimensionless quantity, because λ can then only have the value 1.

*This development is a slightly modified and extended form of one given in lecture notes by Prof. H. Højgaard Jensen and brought to my attention by the late Dr. W. de Groot.

The argument can be easily extended to the case where Y depends on several different non-dimensionless quantities, $P, Q, \dots$. One has:

$$Y = kP^pQ^q \dots. \quad (2.1.13)$$

Leaving open the possibility that k has a physical dimension one can write:

$$\frac{\{Y\}}{\{k\}\{P\}^p\{Q\}^q\dots} = \frac{[k][P]^p[Q]^q\dots}{[Y]} = g \quad (2.1.14)$$

or

$$\{Y\} = g\{k\}\{P\}^p\{Q\}^q\dots \quad (2.1.15)$$

$$[Y] = g^{-1}[k][P]^p[Q]^q\dots. \quad (2.1.16)$$

Here g must be a dimensionless number. It is common practice to choose $g = 1$; the numerical value equation (2.1.15) then has the same form as the quantity equation (2.1.13). It is said that the units for $Y, k, P, Q, \dots$ are coherent with respect to this quantity equation.

When units for the quantities $Y, P, Q, \dots$ are chosen independently the unit of k is fixed by (2.1.16) and the choice of g. When the unit of Y has not been fixed, but the units of $P, Q, \dots$ are known, $[k]$ can be chosen to equal 1, thus making k dimensionless. The unit of Y is now called a derived unit. This was done in (2.1.11), where $g = 1$ and $[k] = 1$. Equation (2.1.16) explains why derived units are found as products of positive and negative powers of base units.

The value of $\{k\}$ can be: fixed by definition, the common choice being $\{k\} = 1$; determined by integration on the basis of definitions already made (see Section 2.2); determined by experiment (see Section 2.2). In the latter case $\{k\}$ is not known exactly.

When in a branch of physics n quantities are introduced between which m relations hold, one has n–m independent or base quantities, each with a base unit. The base units are fixed by definition. All other units are derived ones. The SI is a comprehensive system for science and technology with seven base quantities. All other quantities are derived ones, following from a well-defined set of quantity relations. Their units are coherent with respect to this set, which implies that in all relations of the type (2.1.16) one has $g = 1$.

2.2. *Examples*

(i) Classical mechanics is based on Newton's law:

$$F = ma \quad (2.2.1)$$

with

$$a = \mathrm{d}v/\mathrm{d}t, \quad v = \mathrm{d}s/\mathrm{d}t. \quad (2.2.2)$$

Basic quantities are mass (m), length (s) and time (t), F, v and a are derived quantities defined by (2.2.1) and (2.2.2). In each case $k = 1$. Work performed by F is defined as

$$W = Fs \quad (2.2.3)$$

where again W is a derived quantity.

For the kinetic energy, E, acquired by a mass, m, initially at rest, subjected to a constant force, F, one finds from the preceding equations:

$$E = Fs = m\int_0^t (\mathrm{d}v/\mathrm{d}t)v\,\mathrm{d}t = \tfrac{1}{2}mv^2. \quad (2.2.4)$$

Here $\{k\} = \frac{1}{2}$ arises from integration and $[k] = 1$ because in the SI the unit for both work and kinetic energy is $1\,\mathrm{J} = 1\,\mathrm{N \cdot m} = 1\,\mathrm{kg \cdot m^2 \cdot s^{-2}}$.

(ii) The ideal gas law reads

$$pV = RnT. \quad (2.2.5)$$

The SI units for p and V are $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$ and $1\,\mathrm{m^3}$ respectively, making the unit for pV equal to 1 J. Further, temperature, T, and amount of substance, n, have been chosen as base quantities with units 1 K and 1 mol respectively. Therefore the proportionality factor R has the unit $1\,\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$; its numerical value follows from experiment at 8.314.... An alternative choice could have been to consider temperature as a derived quantity with the unit J/mol. In that case $[R] = 1$; the choice $\{R\} = 1$ would also be possible, leading to a different temperature scale.

This example illustrates that the choice of the number of base quantities is not a fixed one, but determined by convention.

(iii) Empirical relations are found, of course, as relations between numerical values. The good old engineering practice of plotting pairs of experimental values on a log–log graph and drawing a straight line through the points so obtained, finds its physical justification in equation (2.1.12).

It should not be forgotten that dimensionless quantities or groups of quantities form an exception to the rule. Such quantities usually arise from theoretical considerations. Examples are $mv^2/2kT$ in kinetic theory and $h\nu/kT$ in the theory of black body radiation. This may lead to quantity relations of a type different from that expressed by (2.1.12). For example, one finds from Clapeyron's equation for the saturation pressure of low density vapours:

$$\ln\frac{p}{p_0} = \frac{ML}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right), \quad (2.2.6)$$

where M denotes molar mass and L latent heat of vaporization (considered to be constant here). In this case a plot of p on a logarithmic scale against $1/T$ will give a straight line.

It should be noted that $\ln p$ does not make sense, since $\ln Pa$ is meaningless; on the other hand $\ln\{p\}$ does.

(iv) In heat and mass transfer often no closed analytical solutions of the basic equations can be obtained. In that case one determines empirical relations between dimensionless groups of quantities, such as, e.g. Nu, Re and Pr.

An example of such a relation is the following, applied in hot-wire anemometry to express the relation between the hot-wire signal and the wind velocity perpendicular to the wire:

$$Nu = a + bRe^m \quad (2.2.7)$$

where a, b and m are parameters that may depend on temperature and Pr. When measuring in air in a limited temperature range Pr may be considered as constant. From (2.2.7) one finds for the heat-transfer coefficient

$$\alpha = \frac{\lambda a}{d} + \frac{\lambda b}{d}\left(\frac{d}{\nu}\right)^m U^m \quad (2.2.8)$$

where d is the wire diameter, λ the thermal conductivity of the gas and U the wind velocity.

The relation between α and U can be found from calibration measurements and expressed as

$$\alpha = A + B(U/U_r)^m \tag{2.2.9}$$

where U_r is an arbitrary reference velocity. From experiments in our laboratory Koppius [4] found in the temperature range $283\,K < T < 353\,K$ that a best fit could be obtained when A, B and m were all taken to be temperature dependent.

Such a best fit is needed for accurate measurements of turbulent fluctuations of U. A measure of the sensitivity, S, of the hot-wire system to such fluctuations is:

$$S = \frac{\partial\alpha}{\partial U} = \frac{mB}{U_r}\left(\frac{U}{U_r}\right)^{m-1} \tag{2.2.10}$$

The temperature coefficient of S then becomes:

$$\frac{1}{S}\frac{\partial S}{\partial T} = \frac{1}{B}\frac{dB}{dt} + \frac{1}{m}\frac{dm}{dT} + \frac{dm}{dT}\ln\frac{U}{U_r}. \tag{2.2.11}$$

Apparently this quantity depends on the choice of U_r, which may be considered as the unit in which U is expressed.*

The difficulty arises from the fact that m is not a constant, which is at variance with (2.1.12). It is resolved by starting from (2.2.8), which leads to:

$$\frac{1}{S}\frac{\partial S}{\partial T} = \frac{1}{b}\frac{db}{dT} + \frac{1}{\lambda}\frac{d\lambda}{dT} - \frac{m}{v}\frac{dv}{dT} + \frac{1}{m}\frac{dm}{dT} + \frac{dm}{dT}\ln\frac{Ud}{v}. \tag{2.2.12}$$

Although this result is not surprising, the example illustrates that care should be taken in performing mathematical operations on quantity relations that have been derived from experiment.

3. SYMBOLS FOR QUANTITIES IN HEAT AND MASS TRANSFER

The first problem with symbols for quantities arises from the fact that many more kinds of quantities exist than there are letter signs in both the Latin and Greek alphabets, even if one uses upper and lower case, upright, sloping and bold face types. Secondly, strong but often uncoordinated traditions have developed in symbol usage. Such usage may be different for different branches of physics and engineering. Often it arises from the cultural interaction between groups and nations. Language plays an important part in cultural exchange.

The first problem could be mitigated or even solved by the use of less common alphabets and letter types. However, this would make the printing of scientific papers even more expensive and the typewriting of such texts almost impossible. The second problem is under attack by the international organizations for standardization. The present situation is exemplified by the ISO recommendations [2], where often two or more alternative symbols are given for the same quantity, whereas one letter can represent a host of different quantities. This seems inevitable if one wants to cover the whole field of science and technology with the use of the Latin and Greek alphabets only.

Since heat and mass transfer is on the one hand a strongly interdisciplinary field and on the other hand often a highly specialized one, the use of symbols is strongly connected with that in other fields and highly diversified as well. However, I feel that one can and should start to agree on symbols for the quantities that are most frequently encountered. The ISO-recommendations [2], in particular parts I–VI, VIII and XII, can form a starting point for reaching such an agreement.

In our field different sets of symbols are traditionally used by the English speaking workers on the one hand and the continental European ones on the other hand. In the ISO-recommendations these are usually given as equivalent alternatives.

In 1975 the International Centre for Heat and Mass Transfer organized an open forum discussion on the use of symbols, with the aim of agreeing on a single set to be used in a *Heat Exchanger Design and Data Book*, which is under preparation by a committee of experts, chaired by Prof. E. U. Schlünder. Views were expressed in favour of each of the two traditional systems. Nonetheless, it appeared that between those present agreement could be reached to a large extent on a single set of symbols. Among these are α for the heat-transfer coefficient and β for the mass-transfer coefficient. Unfortunately the ISO has dropped in the draft of 31 Part IV [2] the symbol α for the heat-transfer coefficient, which did occur in the previous version, R 31 Part IV. Also in 31 Part XII the symbol k is used for the mass-transfer coefficient.

This shows that the matter of proper symbols for heat and mass transfer is far from resolved. I feel that the heat and mass transfer community should take an active interest in the matter of standardization in its own field, just as was done by Allan Ede, to whose memory this paper is dedicated.

*Without the introduction of U_r the last term in (2.2.11) would have contained $\ln U$, which is meaningless.

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GRANDEURS, UNITES ET SYMBOLES DANS LE TRANSFERT DE CHALEUR ET DE MASSE

Résumé—Le système d'Unités International (S.I.) ne repose pas uniquement sur la définition de sept unités de base, mais aussi sur un ensemble de relations entre grandeurs physiques. Les principes du calcul des grandeurs sont discutés. On montre que la relation entre deux grandeurs arbitraires X et Y doit être de la forme

$$Y = kX^c$$

k et c étant des constantes. Une exception doit être faite pour les quantités adimensionnelles. Des exemples sont donnés.

Maintenant que le système international a été généralement adopté, le moment semble opportun pour qu'un accord intervienne sur l'emploi des symboles relatifs aux grandeurs utilisées dans le transfert de chaleur et de masse. Quelques réflexions sur ce sujet sont présentées.

GRÖSSEN, EINHEITEN UND SYMBOLE IN DER WÄRME- UND STOFFÜBERTRAGUNG

Zusammenfassung—Das internationale Einheitensystem (SI) basiert nicht allein auf den Definitionen der sieben Grundeinheiten, sondern auch auf einem Satz von Beziehungen zwischen physikalischen Größen. Die Grundlagen der Größenberechnung werden diskutiert. Es wird gezeigt, daß die Beziehung zwischen zwei willkürlichen Größen X und Y der Form

$$Y = kX^c$$

genügen muß, wobei k und c Konstanten sind. Eine Ausnahme bilden dabei dimensionslose Kennzahlen. Beispiele werden gegeben.

Nachdem nun das SI-System allgemein akzeptiert ist, scheint die Zeit reif zu sein, eine Übereinstimmung im Gebrauch von Größensymbolen in der Wärme- und Stoffübertragung herbeizuführen. Einige Gedanken hierüber werden dargelegt.

ВЕЛИЧИНЫ, ЕДИНИЦЫ И ОБОЗНАЧЕНИЯ, ИСПОЛЬЗУЕМЫЕ В ТЕПЛО- И МАССООБМЕНЕ

Аннотация — Международная система единиц (СИ) основана не только на определениях семи основных единиц, но также на зависимостях между физическими величинами. Обсуждаются принципы расчета величин. Показано, что зависимость между двумя произвольными величинами X и Y должна иметь вид:

$$Y = kX^c,$$

где k и c постоянны. Исключения составляют безразмерные величины. Приводятся примеры. В настоящее время, когда система СИ в общем признана, наступило время пересмотра обозначений величин и в тепло- и массообмене. Приводятся некоторые соображения по этому вопросу.